Optimal Distillation of Three Qubit W States

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Abstract

Some of the asymmetric three qubit W states are used for perfect teleportation, superdense coding and quantum information splitting. We present the protocols for the optimal distillation of the asymmetric as well as the symmetric W states from a single copy of any three qubit W class pure state.

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I. INTRODUCTION

The use of entanglement as a resource in quantum information and quantum computation requires characterization, manipulation and quantification problems to be solved. The bipartite pure state entanglement has been well understood. In the two qubit case Einstein-Podolski-Rosen (EPR) states $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ are used in many quantum information processes such as perfect teleportation [1] and dense coding [2]. If the initial state used as a resource is not an EPR state but any state in the canonical form $a|00\rangle + b|11\rangle$ $(a \geq b \geq 0)$ then it is possible to perform the task with a maximum probability of success $2b^2$ [3–5]. An alternative way is distilling [6] the EPR state by performing the positive operator valued measurement (POVM) $\frac{b}{a}|0\rangle\langle 0| + |1\rangle\langle 1|$ on the first qubit and then perform the teleportation or dense coding with unit probability. The success probability of the distilling an EPR state turns out to be $2b^2$.

The many body entanglement is not a straight forward generalization of the bipartite case and some challenging problems still remain unsolved. In the three qubit case, for example, three are two classes of tripartite entangled states which can not be converted into each other by stochastic local operations and classical communication (SLOCC) [7], namely the GHZ and W class states. Any two states of the same class can be converted into each other by means of SLOCC. The GHZ state $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and the symmetric W state

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \tag{1}$$

are considered as the representatives of the GHZ and W classes respectively. Although the symmetric W state (1) is more robust against decoherence or particle loses it can not be used to perform perfect quantum information tasks [8, 9]. The asymmetric W states

$$\frac{1}{\sqrt{2}}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|100\rangle, \tag{2}$$

$$\frac{1}{2}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{\sqrt{2}}|100\rangle \tag{3}$$

are widely used in perfect quantum information processes [10–13]. If the three qubit entangled state which will be used as a resource is not a GHZ state or an asymmetric W state (2)-(3) then the distillation these states is necessary to successfully perform quantum information tasks. The optimal distillation of the GHZ state from a single copy of GHZ class state is presented in [14]. Although the distillation of the symmetric W state (1) from some special W states is presented in [15, 16] the protocol for the optimal distillation of the asymmetric W states is still unknown. In this

work we present the protocol for the optimal distillation of the asymmetric W states (2)-(3) as well as the symmetric W state (1) from an arbitrary W class state. The procedure we use is as follows: we first define the canonical form of the W class states which any W class state can be brought into by local unitary transformations and then show that the general local POVMs followed by local unitary transformations which bring the state into canonical form equal to the local POVMs which leave the canonical form invariant. We then find the local POVMs to maximize the probability of obtaining symmetric or asymmetric W states (1)-(3).

II. CANONICAL FORM OF W CLASS STATES AND GENERAL LOCAL POVMS

To define the canonical form of the W class states which we are going to use in the distillation let us first review the canonical form of any three qubit pure state as defined in [17, 18]: any three qubit state

$$|\psi\rangle = \sum_{ijk} t_{ijk} |ijk\rangle \tag{4}$$

defines matrices T_0 and T_1 by

$$|\psi\rangle = \sum_{jk} T_{0,jk} |0\rangle |jk\rangle + T_{1,jk} |1\rangle |jk\rangle .$$

Under the unitary transformations on the first qubit the matrices T_0 and T_1 transform as

$$\begin{aligned} T_0' &= u^A_{00} T_0 + u^A_{01} T_1 \\ T_1' &= u^A_{10} T_0 + u^A_{11} T_1 \quad , \quad u_{tz} = \langle t|U|z\rangle . \end{aligned} \tag{5}$$

It is always possible to make $det T_0' = 0$ and the unitary transformations on the second and third qubits diagonalize T_0'. Then the canonical form of the generic three qubit states is defined by

$$|\psi\rangle = \lambda_0 |000\rangle + \lambda_1 e^{i\varphi} |100\rangle + \lambda_2 |101\rangle + \lambda_3 |110\rangle + \lambda_4 |111\rangle , \quad \lambda_i \geq 0. \tag{6}$$

There are two solutions for $det T_0' = 0$ and hence there are two sets of values of λs for any generic state. If the 3-tangle [19] given by $\lambda_0 \lambda_4$ is nonzero then the three qubit state is of GHZ class. If 3-tangle is zero and the rank of the reduced density matrices $\rho_A \equiv \mathrm{Tr}_{BC} |\psi\rangle \langle\psi|$, ρ_B and ρ_C are two then the states are of W class states. Without loss of generality $\lambda_4 = 0$ and it turns out that there is only one set of λs in (6) for W class states. We also use the fact that the permutation of the parties $1 \leftrightarrow 2$ gives $\lambda_0 \leftrightarrow \lambda_3$, $1 \leftrightarrow 3$ gives $\lambda_0 \leftrightarrow \lambda_2$ and $2 \leftrightarrow 3$ gives $\lambda_2 \leftrightarrow \lambda_3$ and define the canonical form of any three qubit W class state as

$$|\psi\rangle = \lambda_0 |000\rangle + \lambda_1 |100\rangle + \lambda_2 |101\rangle + \lambda_3 |110\rangle \quad (\lambda_0 \geq \lambda_2 \geq \lambda_3). \tag{7}$$

We also note that λ_1 is invariant under the permutation of the parties and the two asymmetric W states (2) and (3) are equal up to the permutation of the parties $1 \leftrightarrow 3$ and hence their canonical forms are equal and given by

$$\frac{1}{\sqrt{2}} |000\rangle + \frac{1}{2} |100\rangle + \frac{1}{2} |101\rangle . \tag{8}$$

The canonical form of the symmetric W state (1) is also found to be

$$|W\rangle = \frac{1}{\sqrt{3}} (|000\rangle + |100\rangle + |101\rangle). \tag{9}$$

We now consider that a general local POVM

$$A' = e^{i\theta_1} a |0\rangle \langle 0| + e^{i\theta_2} b |0\rangle \langle 1| + e^{i\theta_3} c |1\rangle \langle 0| + e^{i\theta_4} d |1\rangle \langle 1| \quad (a, b, c, d \text{ real}) \tag{10}$$

is performed on the first qubit which transforms the state (7) into

$$|\psi'\rangle = \frac{1}{\sqrt{p_A}} (A' \otimes I_B \otimes I_C) |\psi\rangle \tag{11}$$

with probability $p_A = (\langle \psi | (A')^{\dagger} A' \otimes I_B \otimes I_C) |\psi\rangle$ and then the resulting state is brought into the canonical form by local unitary transformations to give

$$|\psi'\rangle = \frac{1}{\sqrt{p_A}} (\lambda_0 \frac{\left| e^{i(\theta_1+\theta_4)} ad - e^{i(\theta_2+\theta_3)} bc \right|}{\sqrt{b^2+d^2}} |000\rangle + \tag{12}$$
$$\left| \lambda_0 \frac{e^{i(\theta_1-\theta_2)} ab + e^{i(\theta_3-\theta_4)} cd}{\sqrt{b^2+d^2}} + \lambda_1 \sqrt{b^2+d^2} \right| |100\rangle + \lambda_2 \sqrt{b^2+d^2} |101\rangle + \lambda_3 \sqrt{b^2+d^2} |110\rangle).$$

Using the fact that the POVM

$$A = \frac{\left| e^{i(\theta_1+\theta_4)} ad - e^{i(\theta_2+\theta_3)} bc \right|}{\sqrt{b^2+d^2}} |0\rangle \langle 0| + \tag{13}$$
$$(\left| \frac{e^{i(\theta_1-\theta_2)} ab + e^{i(\theta_3-\theta_4)} cd}{\sqrt{b^2+d^2}} + \frac{\lambda_1 \sqrt{b^2+d^2}}{\lambda_0} \right| - \frac{\lambda_1 \sqrt{b^2+d^2}}{\lambda_0}) |1\rangle \langle 0| + \sqrt{b^2+d^2} |1\rangle \langle 1|$$

on the first qubit transforms the state (7) into state (12) with the same probability p_A we conclude that the most general POVM on the first qubit is of the form

$$A = a_1 |0\rangle \langle 0| + c_1 |1\rangle \langle 0| + d_1 |1\rangle \langle 1| \quad (a_1, c_1, d_1 \text{ real}). \tag{14}$$

It can similarly be shown that the general local POVMs on the second and third qubits are of the form

$$B = a_2 |0\rangle \langle 0| + b_2 |0\rangle \langle 1| + d_2 |1\rangle \langle 1| \quad (a_2, b_2, d_2 \text{ real}) \tag{15}$$
$$C = a_3 |0\rangle \langle 0| + b_3 |0\rangle \langle 1| + d_3 |1\rangle \langle 1| \quad (a_3, b_3, d_3 \text{ real}).$$

The condition that eigenvalues of $A^\dagger A$, $B^\dagger B$ and $C^\dagger C$ should be less than or equal to one gives the constraints

$$a_i^2 + b_i^2 + d_i^2 + \sqrt{((a_i - d_i)^2 + b_i^2)((a_i + d_i)^2 + b_i^2)} \leq 2, \quad i = 2,3$$
$$a_1^2 + c_1^2 + d_1^2 + \sqrt{((a_1 - d_1)^2 + c_1^2)((a_1 + d_1)^2 + c_1^2)} \leq 2. \tag{16}$$

Hence the most general transformation of the state (7) under local POVMs is given by

$$\begin{aligned} |\psi'\rangle &= \frac{1}{\sqrt{P}} A \otimes B \otimes C |\psi\rangle \\ &= \frac{1}{\sqrt{P}} (\lambda_0 a_1 a_2 a_3 |000\rangle + ((\lambda_0 c_1 + \lambda_1 d_1) a_2 a_3 + \lambda_2 d_1 a_2 b_3 + \lambda_3 d_1 b_2 a_3) |100\rangle \\ &\quad + \lambda_2 d_1 a_2 d_3 |101\rangle + \lambda_3 d_1 d_2 a_3 |110\rangle) \end{aligned} \tag{17}$$

where $P = \langle \psi | A^\dagger A \otimes B^\dagger B \otimes C^\dagger C |\psi\rangle$.

III. OPTIMAL DISTILLATION OF THE ASYMMETRIC W STATES

We now discuss the optimal distillation of the asymmetric W state (8): for

$$\lambda_0 a_1 a_2 a_3 = \sqrt{2} \lambda_2 d_1 a_2 d_3 = \sqrt{2} \lambda_3 d_1 d_2 a_3, (\lambda_0 c_1 + \lambda_1 d_1) a_2 a_3 + \lambda_2 d_1 a_2 b_3 + \lambda_3 d_1 b_2 a_3 = 0 \tag{18}$$

the resulting state is (8) and the probability of success turns out to be

$$P = 2 \lambda_0^2 a_1^2 a_2^2 a_3^2. \tag{19}$$

Now the problem is to find the local POVMs to maximize the probability. The maximization of the local probabilities

$$det(I_A - A^\dagger A) = 0, \; det(I_B - B^\dagger B) = 0, \; det(I_C - C^\dagger C) = 0 \tag{20}$$

implies that the constraints

$$\begin{aligned} (1 - a_1^2)(1 - d_1^2) &= c_1^2, \\ (1 - a_2^2)(1 - d_2^2) &= b_2^2, \\ (1 - a_3^2)(1 - d_3^2) &= b_3^2 \end{aligned} \tag{21}$$

should be satisfied and the state is either transformed into (8) or otherwise disentangled, i.e., we are using *one successful branch protocol* (OSBP). The problem of optimal distillation of the state (8) using OSBP is reduced to the problem of maximizing (19) subject to the constraints (16), (18) and (21). Defining $y \equiv a_3^2$ the maximum probability is found to be the maximum of the function

$$P(y) = \lambda_2^2 + \lambda_0^2 y + \lambda_1^2 y - \lambda_2^2 y + \lambda_3^2 y + \lambda_3^2 y^2 - 2K - \sqrt{L+M}, \;\; (0 < y \leq 1) \tag{22}$$

where

$$\begin{aligned}
K &= \lambda_1 \sqrt{y(1-y)(\lambda_2 - \lambda_3^2 y)}, \\
L &= \lambda_2^4(1-y)^2 + \lambda_0^4 y^2 + \lambda_1^4 y^2 - 2\lambda_1^2\lambda_3^2 y^2 + \lambda_3^4 y^2 + 6\lambda_1^2\lambda_3^2 y^3 + 2\lambda_3^4 y^3 \\
&\quad + \lambda_3^4 y^4 - 4y(\lambda_1^2 - \lambda_3^2 - \lambda_3^2 y)K, \\
M &= 2\lambda_2^2(1-y)(\lambda_0^2 y + 3\lambda_1^2 y + \lambda_3^2 y + \lambda_3^2 y^2 - 2K) + 2\lambda_0^2(\lambda_1^2 y^2 + \lambda_3^2 (y-3) y^2 - 2yK).
\end{aligned} \tag{23}$$

The solutions to the local POVMs are given by

$$\begin{gathered}
a_1 = \sqrt{\frac{P}{2\lambda_0^2 a_3^2}}, \quad d_1 = \frac{\lambda_0}{\sqrt{2}\lambda_3} a_1, \quad c_1 = \sqrt{(1-a_1^2)(1 - \frac{2\lambda_3^2}{\lambda_0^2} a_1^2)}, \\
a_2 = 1, \quad b_2 = 0, \quad d_2 = 1 \\
d_3 = \frac{\lambda_3}{\lambda_2} a_3, \qquad b_3 = \sqrt{(1-a_3^2)(1 - \frac{\lambda_3^2}{\lambda_2^2} a_3^2)}
\end{gathered} \tag{24}$$

where P given by (22) is a function of a_3. We note that the case y=0 ($a_3 = 0$) implies that the rank of the operator C is one, i.e, the third party makes projective measurement which disentangles the third particle from the other two.

To prove that no distillation protocol can give a greater probability one needs to show that the inequality

$$P(|\psi\rangle) \geq \sum_i p_i P(|\psi_i\rangle) \tag{25}$$

is satisfied for any sequence of local quantum operations that transform $|\psi\rangle$ into $|\psi_i\rangle$ with probability p_i. The right hand side of the inequality (25) is the average probability to obtain the state (8) using several branches whereas left hand side is the probability for OSBP. Taking into account that any POVM can be decomposed into a sequence of two-outcome POVMs [14] it is sufficient to show

$$P(|\psi\rangle) \geq p_1 P(|\psi_1\rangle) + p_2 P(|\psi_2\rangle) \tag{26}$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are obtained by the most general POVMs on one of the qubits, say the first qubit. We start with a two outcome POVM with operators

$$A_1 = a_1 |0\rangle\langle 0| + c_1 |1\rangle\langle 0| + d_1 |1\rangle\langle 1|,$$
$$A_2 = \alpha_1 |0\rangle\langle 0| + \gamma_1 |1\rangle\langle 0| + \delta_1 |1\rangle\langle 1| \tag{27}$$

acting on the first qubit satisfying $A_1^\dagger A_1 + A_2^\dagger A_2 = I$. The states

$$|\psi_1\rangle = \frac{1}{\sqrt{p_1}}(\lambda_0 a_1 |000\rangle + (\lambda_0 c_1 + \lambda_1 d_1)|100\rangle + \lambda_2 d_1 |101\rangle + \lambda_3 d_1 |110\rangle),$$
$$|\psi_2\rangle = \frac{1}{\sqrt{p_2}}(\lambda_0 f_1 |000\rangle + (\lambda_0 g_1 + \lambda_1 h_1)|100\rangle + \lambda_2 h_1 |101\rangle + \lambda_3 h_1 |110\rangle) \tag{28}$$

are obtained with probabilities $p_i = \langle\psi| A_i^\dagger A_i |\psi\rangle$. Then OSBP is used on the states $|\psi_1\rangle$ and $|\psi_2\rangle$ to give the maximum probabilities $P(|\psi_1\rangle)$ and $P(|\psi_2\rangle)$ for the distillation of the asymmetric W state (8). To check if the inequality (26) is satisfied we maximize $p_1 P(|\psi_1\rangle) + p_2 P(|\psi_2\rangle)$ and find that the maximum is obtained for $P(|\psi_1\rangle) = 0$ or $P(|\psi_2\rangle) = 0$ which means that no distillation protocol can produce a higher probability of success than the OSBP we present.

The maximization of (22) requires numerical calculations in general. For illustrative purposes we discuss the special case $\lambda_1 = 0$, i.e., the optimal distillation of the state (8) using the W class state

$$|\psi\rangle = \lambda_0 |000\rangle + \lambda_2 |101\rangle + \lambda_3 |110\rangle \quad (\lambda_0 \geq \lambda_2 \geq \lambda_3). \tag{29}$$

For $\lambda_1 = 0$ the maximum of the probability function (22) is given by

$$P = \lambda_0^2 + 2\lambda_3^2 - |\, \lambda_0^2 - 2\lambda_3^2 \,| \tag{30}$$

at the point $y = 1$ ($a_3 = 1$) and the local POVMs turn out to be

$$A = \frac{\sqrt{2}\lambda_3}{\lambda_0}|0\rangle\langle 0| + |1\rangle\langle 1| \quad (\sqrt{2}\lambda_3 \leq \lambda_0), \quad A = |0\rangle\langle 0| + \frac{\lambda_0}{\sqrt{2}\lambda_3}|1\rangle\langle 1| \quad (\sqrt{2}\lambda_3 \geq \lambda_0),$$
$$B = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad C = |0\rangle\langle 0| + \frac{\lambda_3}{\lambda_2}|1\rangle\langle 1|. \tag{31}$$

As an immediate application we consider the teleportation using the symmetric W state as a resource. It is possible to perform the teleportation with unit fidelity but with success probability 2/3 [20] which means that the probability of losing the information is 1/3. However if we prefer not to lose the information to be teleported, we first distill the asymmetric state (8) by the local operations

$$A = |0\rangle\langle 0| + \frac{1}{\sqrt{2}}|1\rangle\langle 1|, \quad B = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad C = |0\rangle\langle 0| + |1\rangle\langle 1| \tag{32}$$

with success probability 2/3 and then o perform the teleportation [4] with unit fidelity and unit success probability. If the distillation is not successful we keep the state to be teleported.

IV. OPTIMAL DISTILLATION OF THE SYMMETRIC W STATES

Our method can also be used for the optimal distillation of the symmetric W state (9) from an arbitrary W class state (7). We again consider the most general local transformations of the W class state (17) and find the local operations which maximize the probability of obtaining the symmetric W state (9). For

$$\lambda_0 a_1 a_2 a_3 = \lambda_2 d_1 a_2 d_3 = \lambda_3 d_1 d_2 a_3, (\lambda_0 c_1 + \lambda_1 d_1) a_2 a_3 + \lambda_2 d_1 a_2 b_3 + \lambda_3 d_1 b_2 a_3 = 0 \tag{33}$$

the resulting state is a symmetric W state (9) with the probability of success

$$P = 3\lambda_0^2 a_1^2 a_2^2 a_3^2. \tag{34}$$

We now maximize (34) under the constraints (16), (21) and (33). Defining $y \equiv a_3^2$ the maximum probability is found to be the maximum of the function

$$P(y) = \frac{3}{2}(\lambda_2^2 + \lambda_0^2 y + \lambda_1^2 y - \lambda_2^2 y + \lambda_3^2 y^2 - 2Q - \sqrt{R+S}), \ \ (0 < y \leq 1) \tag{35}$$

where

$$\begin{aligned} Q &= \lambda_1 \sqrt{y(1-y)(\lambda_2 - \lambda_3^2 y)}, \\ R &= \lambda_2^4 (1-y)^2 + \lambda_0^4 y^2 + \lambda_1^4 y^2 - 4\lambda_1^2 \lambda_3^2 y^2 + 6\lambda_1^2 \lambda_3^2 y^3 + \lambda_3^4 y^4 - 4\lambda_1^2 y Q + 4\lambda_3^2 y^2 Q, \\ S &= 2\lambda_2^2 (1-y)(\lambda_0^2 y + 3\lambda_1^2 y + \lambda_3^2 y^2 - 2Q) + \lambda_0^2 (2\lambda_1^2 y^2 + 2\lambda_3^2 (y-2) y^2 - 4yQ). \end{aligned} \tag{36}$$

and the solutions to the local POVMs are given by

$$\begin{aligned} a_1 = \sqrt{\frac{P}{3\lambda_0^2 a_3^2}}, \quad d_1 = \frac{\lambda_0}{\lambda_3} a_1, \quad c_1 = \sqrt{(1-a_1^2)(1 - \frac{\lambda_3^2}{\lambda_0^2} a_1^2)}, \\ a_2 = 1, \quad b_2 = 0, \quad d_2 = 1 \\ d_3 = \frac{\lambda_3}{\lambda_2} a_3, \qquad b_3 = \sqrt{(1-a_3^2)(1 - \frac{\lambda_3^2}{\lambda_2^2} a_3^2)} \end{aligned} \tag{37}$$

where P given by (35) is a function of a_3. To show that the POVMs given by (37) gives the optimal distillation protocol we need to show that the inequality (26) is not violated by any two outcome

POVM with the operators (27) and the transformed states (28). We find that the maximum of the right hand side of the inequality (26) is obtained for $P(|\psi_1\rangle) = 0$ or $P(|\psi_2\rangle) = 0$. This proves that no distillation protocol can produce a higher probability of success than the OSBP given by (37). For illustrative purposes we again consider the special case $\lambda_1 = 0$, i.e., the optimal distillation of the state (9) from the W class state (29). We find that the maximum of the probability function (35) is $3\lambda_3^2$ at the point $y = 1$ ($a_3 = 1$) and the local POVMs are given by

$$A = \frac{\lambda_3}{\lambda_0} |0\rangle\langle 0| + |1\rangle\langle 1|, \quad B = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad C = |0\rangle\langle 0| + \frac{\lambda_3}{\lambda_2} |1\rangle\langle 1|. \tag{38}$$

V. CONCLUSION

In this work we explicitly constructed the optimal local protocol for the distillation of the symmetric W state (9) as well as the asymmetric W state (8) which is used in perfect teleportation, dense coding and information splitting. We note that in contrast to the distillation of the GHZ states [14] where all three parties should perform local POVMs only two parties should apply POVMs in the distillation of the W states. This is related to the fact that in a general GHZ class state given by (6) three coefficients (λ_1, λ_2 and λ_3) should be made zero which requires the co-operation of all three parties. Since λ_4 is zero for W class states it is possible to distill symmetric or asymmetric W states by the cooperation of only two parties. We have shown that the use of the canonical form and the local POVMs which leave the canonical form invariant simplifies the problem of manipulation of pure states. This result can be used as an alternative approach for the distillation of the GHZ states and the manipulation of other many partite pure states.

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